A SIMPLE SPEADSHEET MODEL TO INCORPORATE SEASONAL GROWTH INTO LENGTH-BASED STOCK ASSESSMENT METHODS

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ABSTRACT

The paper describes a method by which seasonal growth can be incorporated in length-converted catch curves and cohort analysis using a spreadsheet to carry out the analysis. The method is based on calculating the length of fish with the seasonal growth parameters on a daily basis after which a LOOKUP function is used to find length and its corresponding age.

INTRODUCTION

The last decade, fisheries biologists working in tropical waters became increasingly familiar with “length-based fish stock assessment”, mainly due to the development of easy-to-use software, such as LFSA, ELEFAN, FISAT, and LFDA; and good training manuals (Sparre and Venema, 1992); the worldwide FAO/DANIDA training course in Tropical Fish Stock Assessment; and the availability of relatively cheap computers.

Most of the traditional stock assessment methods work with age composition data, whereby “annuli” (on otoliths, scales and other bones) are used to estimate growth. However, in tropical waters this type of “age reading” is almost impossible, and stock assessment took off with the development of “length-based” methods.
In principle, “length-based fish stock assessment” is based on the conversion of length into age, whereby in most cases it is assumed that the fish is growing according to the von Bertalanffy Growth Function (VBGF). It is generally accepted that in temperate waters, the growth of fish displays strong seasonal oscillations, mainly due to fluctuations of temperature and/or food supply (Shul’man 1974). However, strong seasonal growth exists in the tropics (Daget and Ecoutin 1976, de Graaf and Ofori-Danson 1997, de Graaf, 2003). The need to use a seasonal version of the VBGF has been discussed extensively by several authors (Pauly and Ingles 1981; Pauly et al., 1992, Longhurst and Pauly 1987; Sparre, 1990) and seasonal versions of standard analytical models such as the “yield per recruit” method (Sparre, 1991) and length-converted catch curves (Pauly 1990) were developed and incorporated in the different software packages. However, these models can be considered as the basics of stock assessment providing a first idea on the status of the stocks and the impact of fishing mortality. More detailed information, needed for the formulation of fisheries management strategies, is provided through Virtual Population Analysis (VPA) or length-based cohort analysis and Thompson and Bell models (Sparre and Venema, 1992). Pauly et al. (1987) used the seasonal version of the VBGF to slice the cohorts, in a time-based VPA for Peruvian Anchoveta, and this techniques has been incorporated in the DOS version of FISAT, but the length-based VPA in FISAT is still non-seasonal.

Therefore, if you want to analyse data yourself, in a spreadsheet, in most cases you are depending on the mathematics of a non-seasonal model, with distorted results if the growth is seasonal. In this paper, an approach is presented in which seasonal growth can be incorporated in some major fish stock assessment methods using a spreadsheet for analysis.

**NON SEASONAL AND SEASONAL GROWTH**

The basic tool for length-based methods is the conversion of “length-based data” into “age-based data”. For the conversion of length into age in length-based fish stock assessment, traditionally the VBGF is used. The non-seasonal version of VBGF takes the form:

\[
L_t = L_0 \ast \left[ 1 - e^{-K(t-t_0)} \right]
\]


Where

$L_t$  Length at time $t$
$L_\infty$  L infinitive or asymptotic length
$K$  growth parameter
$t_0$  T zero, or time when the fish are born or entered in the system

The growth rate at any point in the lifespan of the fish can be calculated with:

$$\frac{dL}{dt} = K(L_\infty - L_t)$$

Conversion of length into age is being done with the inverse VBGF:

$$t(L) = t_0 - \frac{1}{K} \ln\left(1 - \frac{L}{L_\infty}\right)$$

In conclusion, the conversion of length-based data into age-based data for non-seasonal growth is rather straightforward. However, this is not the case with seasonal growth. The seasonal version of the VBGF (Somers, 1988) has the following form:

$$L(t) = L_\infty \left\{ 1 - e^{-K(t-t_s)-(CK/2\theta)[\sin(2\theta(t-t_s))\sin(2\theta(t_s-t_0))]}} \right\}$$

Where:

$L_t$  Length at time $t$
$L_\infty$  L infinitive
$K$  growth rate parameter
$t_0$  T-zero
$t_s$  the onset of the first oscillation relative to $t=0$, or $t_s = \text{Winter point} + 0.5$
$C$  the intensity of the (sinusoid) growth oscillations

The parameter $C$ is important as it determines the intensity of the seasonal growth. When $C=0$, seasonal growth is absent and the equation equals the standard VBGF. At $C=1$, growth comes to a standstill once a year at the winter-point. Intermediate values of $C$ indicate growth reduction during the winter, but growth never completely stops.
The differences in growth and growth rate (dL/dt) for both versions of the VBGF are illustrated in Figure 1.

\[ \frac{dL}{dt} = -L_\infty \left( -K - CK \cos(2\delta(t - t_s)) \right) e^{-K(t-t_s)-CK\sin(2\delta(t-t_s))} \frac{\sin(2\delta(t-t_s))}{2\delta} \]

However, an inverse seasonal VBGF does not have a direct solution, due to the fact that the right side of the VBGF “t” is found two times in the in the exponential factor, and can only be solved “numerical”.

**ESTIMATING AGE WHEN LENGTH IS KNOWN IN A SEASONAL VBGF.**

Converting length into age for the seasonal VBGF using a numerical mathematical approach would not lead to a simple and practical solution. Therefore, we looked at how we could approach the value of t for a given value of L and the solution is rather easy:

- Calculate the length of the fish with daily intervals for a set of given parameters of the seasonal VBGF
- Search in this data set the length to be converted into age
- Take in this data set the corresponding age value or t
In a spreadsheet, this can be done with a “LOOKUP” function, as explained below.

**STEP 1 CALCULATING LENGTH**

Figure 2 presents the setup of a spreadsheet used to calculate length with seasonal growth. To facilitate building, the VBGF has been separated first into smaller blocks; \(-K(t-t_0); CK/2\delta; \sin^2(t-t_0)\) and \(\sin^2(t-t_0)\), which are combined again in the column cells I10 : I16 to calculate \(L(t)\). In this example from Bangladesh, the fish are born on May 15\(^{\text{th}}\) \((t_o=0.37)\) and have the highest growth in June and a reduced growth in December/January. In the column cells D10: D16, the age of the fish is calculated and together with the column cells I10:I16 they are the basic input for the conversion of length into age. Figure 3 presents the formulae for the different spreadsheet cells. In the example only the first few days’ growth are presented. With the Excel cell copying feature, the formulae in each relevant column can be copied down to row 1104 in order to cover a period of three years.

<table>
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<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
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<td></td>
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<td>K</td>
<td>C</td>
<td>ts</td>
<td>to</td>
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<td>1.3</td>
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<td>0.5</td>
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<td>15 may</td>
<td></td>
<td></td>
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<td><strong>start at “to”</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Day</td>
<td>t</td>
<td>Age</td>
<td>K(t-t_0)</td>
<td>CK/2\delta</td>
<td>\sin^2(2\pi(t-t_\infty))</td>
<td>\sin^2(2\pi(t_\infty-t))</td>
<td>L(t)</td>
<td></td>
</tr>
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<td>0</td>
<td>0.000</td>
<td>0.207</td>
<td>-0.729</td>
<td>-0.729</td>
<td>0.00</td>
<td></td>
</tr>
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<td>0</td>
<td>-0.004</td>
<td>0.207</td>
<td>-0.717</td>
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<td>-0.007</td>
<td>0.207</td>
<td>-0.705</td>
<td>-0.729</td>
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<td>-0.729</td>
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<td>-0.014</td>
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<td>-0.729</td>
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<td>-0.018</td>
<td>0.207</td>
<td>-0.668</td>
<td>-0.729</td>
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<td>-0.021</td>
<td>0.207</td>
<td>-0.655</td>
<td>-0.729</td>
<td>0.47</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 2: Calculating length in a spreadsheet\(^1\)**

\(^1\) In the formula in the spreadsheet \(t_\infty\) or the onset of the first oscillation relative to \(t=0\) is used. Users of FISAT are more familiar with the Winterpoint \(WP=t_\infty+0.5\)
Figure 3: The cell formulae to calculate length with seasonal growth

**STEP 2 INSERTING THE LOOKUP FUNCTION**

With the LOOKUP function in Excel, the instruction is given to search for an indicated value in a column or row, and once the value is found, to provide a corresponding value from another column or row. In our case, LOOKUP searches for a given length in the column where the calculated length data are stored and returns the corresponding age for this length from the column where the ages are stored. The LOOKUP function has three entries:

1. **Lookup_value** is a value that LOOKUP searches for in Lookup_vector and can be a number, a text, or a logical value. In our case it is the \( L(t) \).
2. **Lookup_vector**, is a range that contains only one row or one column of numbers, text or logical values, placed in ascending order. I.e. the column with the "length data".
3. **Result_vector**, is a range that contains only one row or column of number, text or logical values and has the same size as the lookup_vector. I.e. the column with the "age data".

In Figure 4, an example of conversion of length into age with the LOOKUP function is presented. In cells M10:M22 and N10:N22 the length classes, with a 1 cm interval, are entered, and the mid-length of each length class is calculated in the cells O10:O22. With the LOOKUP function, entered in cells P10:P22, the age of each mid-length is estimated in the spreadsheet. In cell O10, the mid-length of the interval 0-1 cm is calculated (0.5 cm), and with the LOOKUP function the column L is searched for the first value approaching 0.5. Once found, in cell L16, it takes the corresponding age value from column K and enters this value in cell P10. In our example the first value, approaching 0.5 cm, is 0.469 cm on day 6, with an age of...
0.016 year. It could be argued that the method is rather inaccurate. However, in the example, the fish are growing with steps of one day. Reducing this to steps of 0.25 day improves the accuracy drastically, but also increases the size of the spreadsheet.

In Figure 5 the formulae for the different cells are presented:

Figure 4: Converting length into age with the LOOKUP function

Figure 5: The cell formulae for the conversion of length into age with a LOOKUP function.
Length-based methods are often used when length composition data for the total fishery is available for a one-year period only. This is often the case in fish stock assessment programmes in developing countries where the funds for continuous fishery monitoring are lacking. Catch curves and cohort analysis can be applied under these conditions as their basic assumption is the constant parameter system: The picture presented by all length classes caught during one year reflects that of a single cohort during its entire life span (Sparre and Venema, 1992). In the next paragraphs it will be demonstrated how our method can be used for the construction of a linearized length-converted catch curve and cohort analysis when growth is seasonal.

**LINEARIZED LENGTH-CONVERTED CATCH CURVE**

The construction of a catch curve is the most common approach to estimate the total mortality of a cohort. Details behind the method are well presented by Sparre and Venema (1992) and Gayanilo and Pauly (1997) and are only briefly summarized here. Assuming constant recruitment and constant mortality the length converted catch curves takes the form:

$$\ln\left(\frac{C_i}{dt_i}\right) = a + Zt'_i$$

Where
- $C_i$ catch number of length class $i$
- $dt_i$ time needed of the fish to grow through length class $i$
- $Z$ Total mortality
- $t'_i$ age of the mid-length of length class $i$
- $a$ constant

For non-seasonal growth $dt_i$ is estimated from

$$dt_i = \left(\frac{1}{K}\right)\ln\left[\frac{L_{i+1}}{L_{i+1} - L_i} \right]$$

and $t'_i$ is estimated from

$$t'_i = \left(\frac{1}{K}\right)\ln\left[1 - \left(\frac{L_i}{L_{i+1}}\right)\right]$$
Sparre (1990) and Pauly (1990) clearly demonstrated that the total mortality is overestimated with a traditional catch curve, if seasonal growth is not accounted for. The reason is that $dt_i$ and $t'_i$ depend not only on length but also on the time of the year if growth is seasonal. Pauly (1990) developed a method using the parameters of the seasonal VBGF to identify a number of pseudo cohorts to resolve this problem, which is incorporated in FISAT. It is rather complicated to apply this method in a spreadsheet; therefore, we explain how a catch curve can be made with the LOOKUP function, providing results comparable with the method of Pauly. We illustrate this with an example of *Puntius sophore* from Bangladesh.

In Table 1 the number of *Puntius sophore* caught in a one-year stock assessment program for the different length classes is presented. The distribution is believed to be representative for the overall population structure as the fish were caught with non-selective gears.

<table>
<thead>
<tr>
<th>L1 (Cm)</th>
<th>L2 (Cm)</th>
<th>Mid length (Cm)</th>
<th>Catch numbers</th>
<th>Growth parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.0</td>
<td>0.5</td>
<td>10</td>
<td>Linf 13</td>
</tr>
<tr>
<td>1.0</td>
<td>2.0</td>
<td>1.5</td>
<td>250</td>
<td>K 1.3</td>
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<tr>
<td>2.0</td>
<td>3.0</td>
<td>2.5</td>
<td>590</td>
<td>to 0.37</td>
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<tr>
<td>3.0</td>
<td>4.0</td>
<td>3.5</td>
<td>70</td>
<td>ts 0.5</td>
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<td>4.0</td>
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<td>C 1</td>
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<td>12.0</td>
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</table>

**Table 1:** Length frequency distribution and estimated growth parameters of *Puntius sophore* as obtained through a one-year stock assessment program

In order to construct the catch curve, we have to calculate $dt_i$, the time needed for the fish to grow through length class $i$. For each length class this is done with the LOOKUP function as:
\[ dt = \text{age L2} - \text{age L1} \]

Conversion of the mid-length into age has been demonstrated in the previous paragraph, and construction of the catch curve becomes straightforward and is presented in Figure 6 and Figure 7.

<table>
<thead>
<tr>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>O</th>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>S</th>
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<td>17</td>
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<td>S</td>
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<td>L1</td>
<td>L2</td>
<td>Mid length</td>
<td>Age of mid length</td>
<td>Age L1</td>
<td>Age L2</td>
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**Figure 6: Constructing a seasonalised catch curve**
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<th>R</th>
<th>S</th>
<th>T</th>
<th>U</th>
<th>V</th>
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<tr>
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<td></td>
<td>Figure 7: The cell formulae for the seasonalised length-converted catch curve</td>
<td></td>
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</tr>
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<td>10</td>
<td>Age</td>
<td>L1</td>
<td>L2</td>
<td>Mid length</td>
<td>Age of mid length</td>
<td>lookup</td>
<td>Age 1.1</td>
<td>lookup</td>
<td>Age 1.2</td>
<td>df</td>
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<td>+1</td>
<td>3</td>
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</tr>
<tr>
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<td>-N17</td>
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<td>1150</td>
<td>-N18+N18</td>
<td>-N18+N18</td>
</tr>
</tbody>
</table>
In column S, delta t (dt) is calculated and ln(C/dt) is calculated in column T. Plotting Ln(C/dt) against the age of the mid-length (column P) gives the catch curve (Figure 8) indicating a total mortality of 4.57 year\(^{-1}\), which is similar with the results of analysing the data in FISAT with the method of Pauly (1990).

**Figure 8: Results of length-converted catch curve of *Puntius sophore*, made in a spreadsheet (A) and with FISAT (B)**

**COHORT OR VIRTUAL POPULATION ANALYSIS\(^2\)**

Cohort or Virtual Population Analysis use the number of fish caught during commercial fishing operations to estimate historic fishing mortality and stock numbers in a cohort of fish and is again based on the constant parameter system.

The number of fish surviving from one year \((N_i)\) to the next year \((N_{i+1})\) is given by:

\[
N_{i+1} = N_i \cdot e^{[-(F_i+M)]}
\]

The number of dying fish is therefore:

---

\(^2\) Adapted from Gayanilo and Pauly (1997) and King (1995)
The catch \((C_t)\) is the proportion dying owing to fishing, and may be estimated from
the catch or Baranov (1926) equation;

\[
C_t = \left(\frac{F_t}{Z_t}\right) \cdot N_t \cdot \left(1 - e^{-\frac{(F_t + M)}{Z_t}}\right)
\]

Combining the different equations will give the Gulland (1965) equation for Virtual
Population Analysis:

\[
\frac{C_t}{N_{t+1}} = \left(\frac{F_t}{Z_t}\right) \cdot e^{Z_t - 1}
\]

Given values of the catch \((C_t)\), and an estimate of the natural mortality \(M\), the
equation can be used to estimate retroactively the size of the past cohorts, if an
estimate of \(N_{t+1}\) is available from which to start the computation. Estimates of \(N_{t+1}\)
(expressing the last population size a cohort had before it became extinct) are called
“terminal population” \(N_t\). Values of \(N_t\) can be obtained from:

\[
N_t = Z_t \cdot \frac{C_t}{F_t}
\]

Where \(C_t\) is the terminal catch (i.e. the last catch taken from a cohort before it went
extinct) and \(F_t\) is the terminal fishing mortality, i.e. the fishing pressure that generated
\(C_t\). A VPA starts with an initial guess of \(F_t\) and then calculates backwards with the
known catches and natural mortality rate (Figure 9):
The major aim of cohort analysis is to estimate the fishing mortality ($F$) over the different length classes. The basics on how to do it are well explained by Sparre and Venema (1992) and only summarized here. The basic formulae for length based cohort analysis are:

$$N(L1) = \left[ N(L2), e^{\left(\frac{M \cdot dt}{2}\right)} + C(L1,L2) \right] e^{\left(\frac{M \cdot dt}{2}\right)}$$

and

$$C(L1,L2) = \frac{F}{Z} \left[ 1 - e^{-Z \cdot dt} \right]$$

Where

- $C(L1,L2)$ the number of fish caught of length between $L1$ and $L2$
- $N(L1)$ the number of fish that attain length $L1$
- $dt$ time needed for the fish to grow through length $L1$ to length $L2$
- $M$ natural mortality during time $dt$
- $F$ fishing mortality during time $dt$
- $Z$ total mortality during time $dt$

In a cohort analyses with non-seasonal growth $e^{\left(\frac{M \cdot dt}{2}\right)}$, the fraction of $N(L1)$ that survives natural death during the time period from $t(L1)$ to $t(L2)$ and $dt$ is calculated as:

$$e^{\left(\frac{M \cdot dt}{2}\right)} = \left[ \frac{L\infty - L1}{L\infty - L2} \right] ^{\frac{M}{2K}}$$

and

$$dt = \left( \frac{1}{K} \right) \ln \left[ \frac{L\infty - L1}{L\infty - L2} \right]$$
Again with seasonal growth the last formulae will give incorrect results. However, \( dt \) can be calculated as: \( dt = \text{age } L_2 - \text{age } L_1 \), which can be solved with the LOOKUP function and thus we can use the basic formulae \( \exp \left( \frac{M \cdot dt}{2} \right) \) directly.

In Figure 10 an example of a cohort analysis in a spreadsheet for *Puntius sophore* with a natural mortality of \( M=1.168 \text{ year}^{-1} \) (all other parameters are the same as those in the previous examples) is presented; the cell formulae are presented in Figure 11.
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<th>Age</th>
<th>t (t)</th>
<th>L1</th>
<th>L2</th>
<th>mid length</th>
<th>Age</th>
<th>y(t)</th>
<th>dy</th>
<th>Catch</th>
<th>exp(M*t/f)(t)</th>
<th>M1=(M1<em>exp(M1</em>t/f)*Catch</th>
<th>exp(M1<em>t/F)<em>F2=F</em>Catch</em>exp(M1*t/F)*M1</th>
<th>F=F<em>exp(M1</em>t/F)</th>
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**Figure 10:** Example of cohort analysis with seasonal growth in a spreadsheet
Figure 11: Cell formulae for cohort analysis with seasonal growth
In Figure 12 the fishing mortality (F) for the different length classes, calculated with non-seasonal and seasonal growth, is presented. The comparison indicates that the traditional non-seasonal method under-estimates F during the period of slow growth, which in our example is occurring at a length of 8-9 cm. The main reason is that the fish stay for a long period in this length class, while fishing continues. The difference seems to be subtle, but the practical consequences can be large. For example in Bangladesh, growth reduces during the dry season (December-January). During this period the floodplains are small as they are drying up; consequently, the fishing effort is high. Underestimating the fishing mortality during this period will have serious consequences if the estimates are used in Thompson and Bell models for the comparison of fisheries management options.

In this respect, it is important to notice that the LOOKUP function can be used in a similar way to calculate “dt” for seasonal growth in Thompson and Bell models and in the seasonal version of the Yield per Recruit Analyses (Sparre, 1991).

![Figure 12: Results of a cohort analysis with seasonal growth and the fishing mortality as estimated by using the seasonal or non-seasonal VBGF.](image)
LIMITATION OF THE PROPOSED METHOD

The proposed method provides convenient results but has some limitations. First of all, the time of recruitment should be known. This is a minor limitation as in most cases it is known during which month major spawning takes place.

Secondly, the used seasonal version of the VBGF of Somers (1988) has exactly one zero growth rate per year when C=1, which means that for each length there will always be one value for its age. Using this version of the VBGF will do for most tropical fisheries, where prolonged periods of zero growth will be an exception. The method cannot be applied if the seasonal version of the VBGF of Pauly et al (1992) is used as this version allows for longer periods of “no growth”, which means that a length can have several values for age, i.e. we cannot convert length into age.

A similar problem arises if there are two cohorts per year, which is often the case in penaeid shrimps. Then again, there is no “one-to-one” correspondence between age and length (Sparre 1990). In this case, the only solution is to slice the cohorts (Sparre and Venema, 1998) and apply a VPA with pseudo cohorts (Pauly et al. 1987, Gayanilo and Pauly, 1997)

THE SPREADSHEETS

The different spreadsheets can be downloaded from our website www.nefisco.org/Training.htm

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REFERENCES


